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DETERMINATION OF CONDENSATE LAYER THICKNESS IN THE INTERACTION OF A HIGH-TEMPERATURE TWO-PHASE FLOW WITH A WALL

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Calculation algorithms are developed that permit a determination of the time variation for the position of the outer boundary of a condensate film, depositing on a barrier, based on its surface temperature measured experimentally.

As a result of the interaction of a high-temperature gas flow containing liquid particles with a barrier, whose temperature is lower than the melting temperature of the particles, a solidified condensate layer is formed on it. The effect of the condensate layer deposition is manifested, on the one hand, in the enhancement of the thermal action due to direct energy transfer from the condensed phase to the barrier and, on the other hand, in the formation of a kind of a "coating" possessing a certain specific heat and thermal resistance.

The heat flux coming onto the barrier when a solidified condensate layer is formed on its surface is largely dependent on the rate of the layer build-up. In experimental studies of the rate of the heat exchange of the barrier with the high-temperature gas flow containing liquid particles direct measurement of the build-up rate of the solidifying condensate layer is unfeasible, and only nonstationary temperatures of the barrier surface can be measured with the aid of sensors of the wall temperature.

Information on the surface temperature of the barrier allows a reconstruction of the incoming heat flux. By means of these data it is possible, in turn, to formulate an inverse problem of heat conduction that consists in determining the law of motion for the phase transition front with a constant temperature equal to the melting temperature of the deposit material. (It is assumed that a condensate layer with temperature of the outer surface not higher than the melting temperature maintains itself on the barrier; liquid particles that have not managed to cool on the barrier are entrained by the gas flow). Such a problem in terms of inverse heat conduction problems may be included among so-called geometric statements. The solution of this problem reduces to solving a system of nonlinear Volterra integral equations of the first kind. This is typically an incorrect problem [1, 2] whose solution is complicated by nonlinearity relative to the sought parameter. We consider it reasonable to attempt to construct the algorithms for calculating the boundary position on the basis of the solution of the inverse boundary-value problem of heat conduction for a "fictitious" wall with thickness specified a priori. Such an approach suggests that the corresponding thermal conditions at the moving boundary of the actual problem may be reproduced adequately by "selecting" the conditions at the outer surface of the fictitious wall and the initial temperature distribution across its thickness. Such an approach may be validated by analyzing the well-known analytic solution of the Stefan problem [3] on freezing of a semi-infinite mass, where the position of the front of phase transitions (the isotherm position) may actually be found by considering the equivalent problem of heat conduction with no phase transitions for a wall with a thickness prescribed a priori and an initial temperature that is a function of the problem parameters.

With a view to the above considerations, the problem in a one-dimensional statement may be formulated as follows.

An infinite plate of thickness δ_2 (2), thermally insulated on one side, is brought in contact at the initial moment with a condensate layer of fictitious thickness δ (1), in which, under the effect of certain conditions at the outer boundary, temperature fields with a moving isotherm corresponding to the temperature of the phase transition

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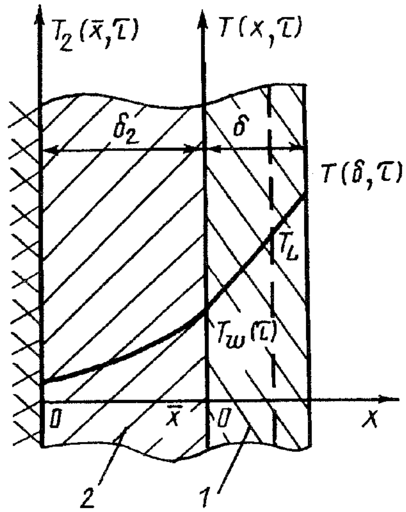

Fig. 1. Thermal model of the problem.

are realized (Fig. 1). Information on the conditions at the fictitious boundary is obtained from solving the following incorrect problem:

$$\frac{\partial^2 T(x, \tau)}{\partial x^2} = \frac{1}{a} \frac{\partial T(x, \tau)}{\partial \tau}, \quad 0 \le x \le \delta; \tag{1}$$

$$\lambda \frac{\partial T(x, \tau)}{\partial x}\Big|_{x=0} = q_w(\tau); \tag{2}$$

$$T(0, \tau) = T_w(\tau); \tag{3}$$

$$T(x, 0) = u(x) = ? \tag{4}$$

$$T(\delta, \tau) = ? \tag{5}$$

where $T_w(\tau)$ is a function known from experiment, and $q_w(\tau)$ can be found by recalculating the boundary conditions on the basis of the solution to the problem:

$$\frac{\partial^2 T_2(\bar{x}, \tau)}{\partial \bar{x}^2} = \frac{1}{a_2} \frac{\partial T_2(\bar{x}, \tau)}{\partial \tau}; \quad 0 \le \bar{x} \le \delta_2; \tag{6}$$

$$T_2(\delta_2, \tau) = T_w(\tau); \tag{7}$$

$$\frac{\partial T_2(\bar{x}, \tau)}{\partial \bar{x}}\Big|_{\bar{x}=0} = 0; \tag{8}$$

$$T_2(\bar{x}, 0) = T_0^*; \tag{9}$$

$$\lambda_2 \frac{\partial T_2(\bar{x}, \tau)}{\partial \bar{x}}\Big|_{\bar{x}=\delta_2} = q_w(\tau) = ? \tag{10}$$

The solution of the systems of equations (6)-(9) and (1)-(4) has the form

$$q_w(\tau) = \frac{\lambda_2}{\delta_2} \int_0^\tau \bar{T}'_w(\theta)\, k(\tau - \theta)\, d\theta ; \tag{11}$$

$$\int_0^\tau \bar{T}'(\delta, \theta)\, \tilde{k}(\tau - \theta)\, d\theta = \bar{T}(0, \tau) + \frac{1}{\sqrt{\lambda c \rho}} \int_0^\tau q_w(\theta)\, \bar{\bar{k}}(\tau - \theta)\, d\theta -$$

$$- \bar{T}(0, 0)\,[1 - \bar{R}_1] - \sum_{i=1}^{m-1} \bar{T}(i\Delta x, 0)\,[\bar{R}_i - \bar{R}_{i+1}] - \bar{T}(\delta, 0)\,[\bar{R}_m - \bar{R}_\delta], \tag{12}$$

Converting from the integral representation (11) and (12) to a numerical approximation, we write the following matrix expressions:

$$\mathbf{q}_w = \frac{\lambda_2}{\Delta\,\mathrm{Fo}_2\,\delta_2} \bar{k}\, \bar{\mathbf{T}}(0) ; \tag{11'}$$

$$\tilde{\tilde{k}}\, \bar{\mathbf{T}}(\delta) = \left(1 - \frac{\delta}{\lambda} \frac{\lambda_2}{\Delta\,\mathrm{Fo}_2\,\delta_2} \bar{\bar{k}}^*\, \bar{k}\right) \bar{\mathbf{T}}(0) - B\bar{\mathbf{T}}_0 , \tag{12'}$$

where $\mathbf{q}_w$, $\bar{\mathbf{T}}(\delta)$, and $\bar{\mathbf{T}}_0$ are vectors whose elements correspond to $q_{w}(\tau)$, $\bar{T}(0,\tau)$, $\bar{T}(\delta, \tau)$, and $\bar{T}(x, 0)$ at the discretization points; $\Delta\tau$ and $\Delta(x)$ are the steps of discretization in time and along the coordinate; $\Delta\mathrm{Fo}_2 = (a_2\Delta\tau)/\delta_2^2$; $\Delta\mathrm{Fo} = (a\Delta\tau)/\delta^2$; and I is the unit matrix of N-th order.

The elements of the matrices $\bar{k}$, $\tilde{\tilde{k}}$, $\bar{\bar{k}}^*$, and B are determined using the expressions

$$\bar{k}(n\Delta\tau) = 1 - \sum_{k=1}^{\infty} \frac{8}{\pi^2 (2k-1)^2} \exp[-\mu_k^{*2}\, n\Delta\,\mathrm{Fo}_2] ;$$

$$\tilde{\tilde{k}}(n\Delta\tau) = n\Delta\,\mathrm{Fo} - \frac{1}{2} + \frac{16}{\pi^3} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k-1)^3} \exp[-\mu_k^{*2}\, n\Delta\,\mathrm{Fo}] ;$$

$$\bar{\bar{k}}^*(n\Delta\tau) = 1 - \sum_{k=1}^{\infty} \frac{2}{\mu_k^{*2}} \exp[-\mu_k^{*2}\, n\Delta\mathrm{Fo}] ;$$

$$\bar{R}_i(n\Delta\tau) = 1 - \sum_{k=1}^{\infty} 2\frac{(-1)^k \cos\left[\mu_k^*\left(1 - \frac{2i-1}{2}\frac{\Delta x}{\delta}\right)\right]}{\mu_k^*} \exp[-\mu_k^{*2}\, n\Delta\,\mathrm{Fo}] ;$$

$$\bar{R}_\delta(n\Delta\tau) = 1 - \sum_{k=1}^{\infty} 2\frac{(-1)^k}{\mu_k^*} \exp[-\mu_k^{*2}\, n\Delta\,\mathrm{Fo}] ,$$

where $\mu_k^* = (2k - 1)\pi/2$.

As is known [4], an exact solution of Eq. (12) can be obtained only for a function $T_w(\tau)$ that satisfies very stringent conditions. Should an error be imposed on this function, Eq. (12) can no longer be solved exactly, generally speaking. It has to be solved approximately with the use (by virtue of the random character of the errors) of statistical considerations resembling those used in the theory of signal separation on a background of random interferences (noises).

Allowance for disturbances and their removal permit an effective solution of the formulated problem. The elements of the matrix $\tilde{\tilde{k}}$ are represented as

$$\tilde{\tilde{k}}_r s = \sum_{j=1}^{N} \sqrt{\mu_j}\, \psi 1_j (r)\, \psi 2_j (s),$$

where the functions $\psi 1_j$ and $\psi 2_j$ are eigenvectors and μ_j are eigenvalues of the matrices

$$k1 = \tilde{\tilde{k}}\, \tilde{\tilde{k}}^T, \quad k2 = \tilde{\tilde{k}}^T\, \tilde{\tilde{k}}.$$

Then the solution of Eq. (12′) has the form

$$\hat{\bar{T}}_i (\delta) = \sum_{j=1}^{N} \frac{c_j}{\sqrt{\mu_j}\,(1 + t_j)} \psi 2j (i), \tag{13}$$

where $c = \psi 1^T z$; z is the right side of Eq. (12′); and $t_j = \overline{\delta c_j^2}/\bar{c}_j^2$ is the "interference-signal" ratio.

Instead of the unknown $\bar{c}_j$ use should be made of

$$\hat{c}_j = c_j \omega_j,$$

where $\omega_j = \frac{1}{2} + (1/4 - \overline{\delta c_j^2}/c_j^2)^{1/2}$ is the weight function.

Ultimately, the expression for the temperature at the fictitious boundary has the matrix form

$$\hat{\bar{\mathrm{T}}} (\delta) = \psi 2\mu_\omega \psi 1^T\, W \bar{\mathrm{T}} (0) - \psi 2\mu_\omega \psi 1^T\, B \bar{\mathrm{T}}_0 . \tag{14}$$

It involves the unknown vector of the initial temperature distribution in the fictitious layer $\bar{\mathrm{T}}_0$, which is found from the following reasoning.

Because solution (14) is approximate, the heat flux at the inner surface of the fictitious layer

$$\hat{\mathrm{q}} (0) = - P_0 \bar{\mathrm{T}} (0) + P_\delta \hat{\bar{\mathrm{T}}} (\delta) + R\bar{\mathrm{T}}_0 \tag{15}$$

calculated with its aid does not coincide with the heat flux at the same point predicted from Eq. (11′).

On substituting expression (14) into Eq. (15) and taking account of Eq. (11′), we set up the functional

$$\sum_{n=1}^{N} [q_{w_n} - \hat{q}_n (0)]^2 = A . \tag{16}$$

To find the elements of the vector $\bar{\mathrm{T}}_0$, we minimize functional (16) by a gradient method using the minimizing sequence

$$\bar{T}_{0_i}^{(j+1)} = \bar{T}_{0_i}^{(j)} - \frac{G_i}{|G_i|} \frac{\| G \|^2 - G_i^2}{\| G \|^2} \varepsilon_{T_0} \bar{T}_{0_i}^{(j)}, \tag{17}$$

where

$$\mathrm{G} = - 2 (P_\delta \psi 2\bar{\mu}_\omega \psi 1^T B + R)^T \times$$

$$\times \left\{\mathrm{q_w} + [P_0 - P_\delta \psi 2\mu_\omega \psi 1^T W]\, \bar{\mathrm{T}} (0) + (P_\delta \psi 2\mu_\omega \psi 1^T B - R)\, \bar{\mathrm{T}}_0\right\}$$

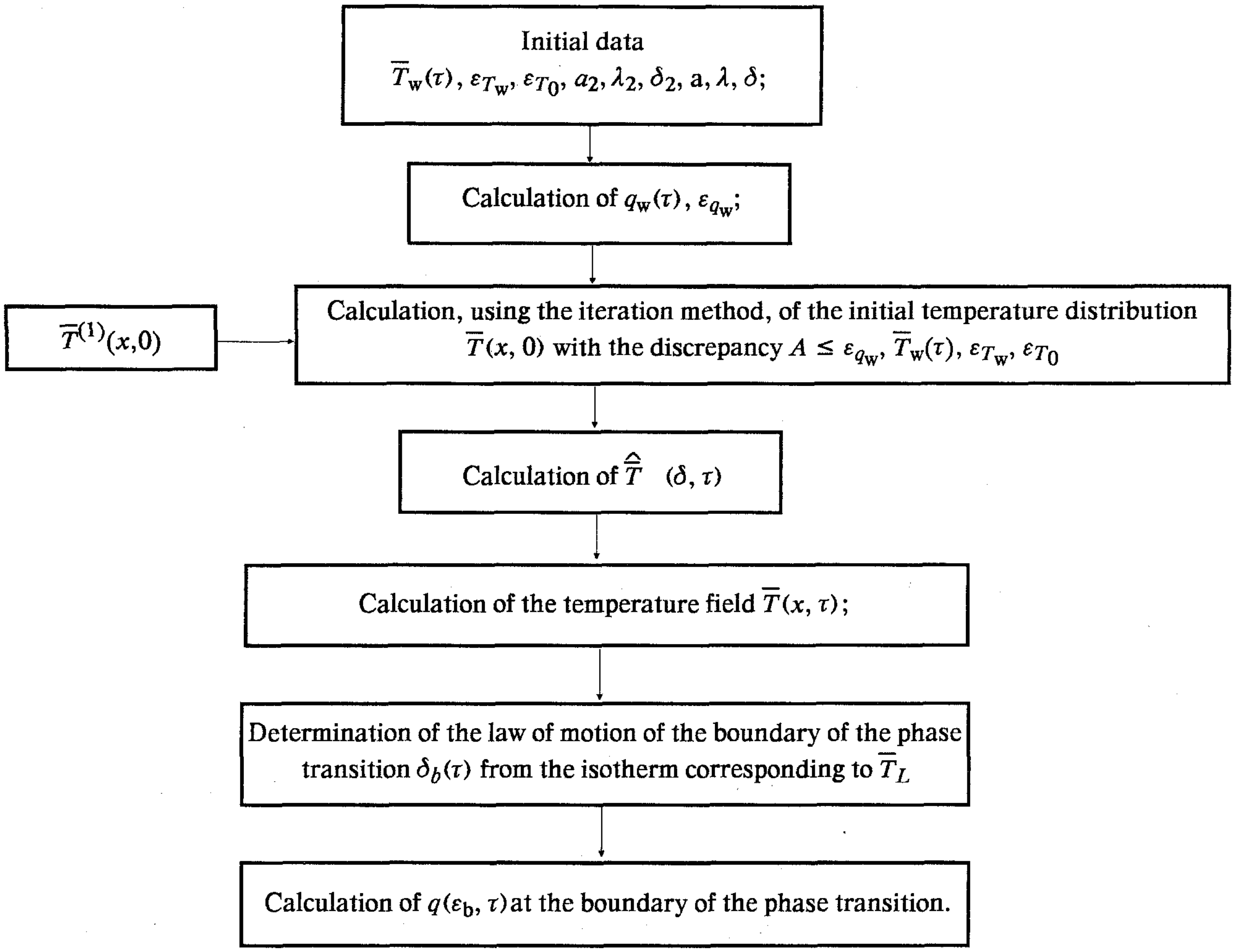

Fig. 2. Block diagram of the calculation program for the thickness of the condensate layer and the density of the heat flux on its outer boundary.

is the vector of the gradient of functional (16) with respect to the corresponding elements of the vector $\overline{T}_0$.

The iteration is continued until the discrepancy condition $A \le \varepsilon_{q_w}$ is fulfilled, where ε_{q_w} is the computational error for the heat flux.

The elements of the matrices P_0, P_δ, and R are determined using the expressions

$$P_i(n\Delta\tau) = \frac{\lambda}{\delta}\Bigg\{1 - 2\sum_{k=1}^{\infty}(-1)^{k+1} \times$$

$$\times \cos\left[\pi k\left(1 - \frac{2i-1}{2}\frac{\Delta x}{\delta}\right)\right]\exp(-\pi^2 k^2 n\Delta\,\mathrm{Fo})\Bigg\};$$

$$P_0(n\Delta\tau) = \frac{\lambda}{\delta}\left[1 + 2\sum_{k=1}^{\infty}\exp(-\pi^2 k^2 n\Delta\,\mathrm{Fo})\right];$$

$$P_\delta(n\Delta\tau) = \frac{\lambda}{\delta}\left[1 - 2\sum_{k=1}^{\infty}(-1)^{k+1}\exp(-\pi^2 k^2 n\Delta\,\mathrm{Fo})\right].$$

Having determined the initial temperature distribution in the fictitious layer $\overline{T}(x, 0)$, we find the temperature at the fictitious boundary $\overline{T}(\delta, \tau)$ from Eq. (14) and, based on the solution of the direct problem of

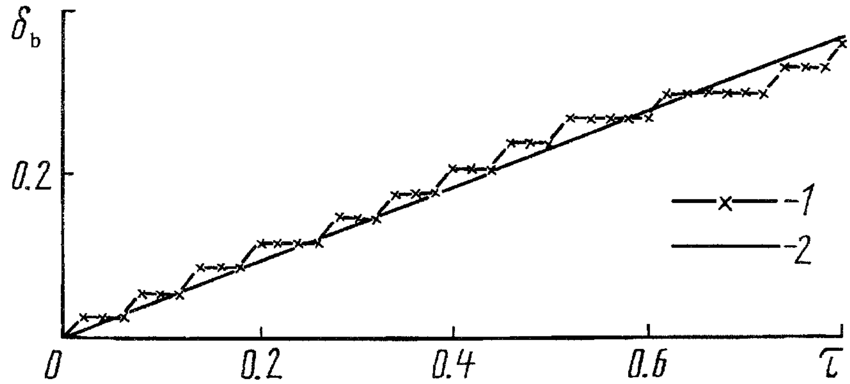

Fig. 3. Results of a numerical experiment: 1) evaluation of the position of the phase transition boundary by the proposed algorithm (ε_{T_w} = 0.05; ε_{T_0} = 0.05, and δ = 1.5); 2) data of the analytic solution. δ_b, mm; τ, sec.

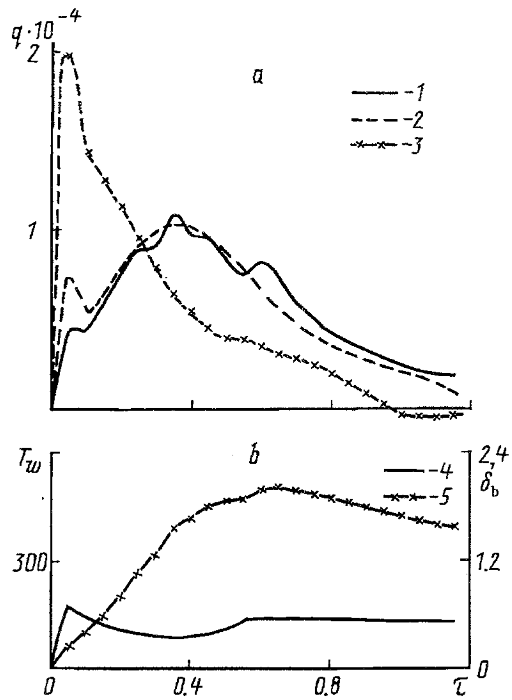

Fig. 4. Results of processing the data of a physical experiment: 1) q_w is the heat flux density on the barrier calculated from T_w, λ, and a of the barrier; 2) b_{wp} is the evaluation of the heat flux density on the barrier calculated from T_w, q_{δ_b} , δ_b, λ and a of the condensate layer; 3) q_{δ_b} is the calculated heat flux density on the outer boundary of the condensate layer; 4) δ_b is the thickness of the condensate layer; 5) T_w is the measured surface temperature of the barrier. q, kW·m^{-2}; T_w, °C; δ_b, mm.

heat condition, determine the temperature field at discrete points of the fictitious domain, which permits isolation of the coordinate of the isotherm corresponding to the melting temperature of the condensate. The time dependence of the coordinate $\delta_b(\tau)$ is the law of motion of the boundary of the phase transition during condensate deposition on the barrier.

With regard for the above, we developed a program for the computer calculation (see the block diagram in Fig. 2) of the thickness of the condensate layer on the barrier $\delta_b(\tau)$ and the heat flux density q (δ_b, τ) at the outer boundary of the layer. The program developed was checked by a numerical experiment that solved a problem of freezing of an infinite mass that has an analytic solution [3].

Analysis of the results of the numerical experiment indicated satisfactory convergence of the proposed and exact solutions (Fig. 3).

Using the proposed procedure, we processed results of a physical experiment with formation of a condesate film on the surface of a sensor of wall temperature. The processing data are given in Fig. 4.

NOTATION

λ, thermal conductivity; a, thermal diffusivity; x, $\bar{x}$, spatial coordinates; T, temperature; τ, time.

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